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Correspondence

Mean power or rms power?

I was most interested to read my latest issue Vol. 33 No. 4 of Electronic Applications Bulletin; as usual it maintained the established high standard of its predecessors. However I was disappointed to see in the article *Audio Power Amplifiers with Darlington Output Transistors* the last words on page 157 'Continuous rms power . . .'. This was all the more disappointing when it occurs in a journal with the professional reputation that your journal rightly enjoys and in an area where precision in terminology might be expected, that is, the section labelled *Definition of terms*. It is clear from the context that the authors intended 'mean power' and I can see no reason why mean power should not be used.

I have taken the opportunity to enclose a note I give to my students *(opposite)*. It suffers somewhat from overkill but this is often necessary for students.

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A note on the word power

Power is defined in electrical circuits as the product of current flowing in the circuit and the voltage drop across the circuit. Should the current and voltage be functions of time then in general so will power be a function of time. In many situations we are not interested in the short-term variation of power but rather in its mean effect over a relatively long time interval. For example semiconductors will fail if the junction temperature exceeds some fairly critical level. Since the thermal capacity of such things is usually sufficiently large, the junction temperature will not change appreciably over a cycle or so of the power fluctuations. Hence, what is significant in these conditions is the *mean* power over a time interval substantially greater than the thermal time constant of the system.

Mean power is defined just like any other mean, i.e.

$$\text{mean power} = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t \qquad (1)$$

where $p(t)$ is the instantaneous power at time t and T is the time interval over which the mean is to be taken. In an electrical circuit power may be expressed as

$$p(t) = \{i(t)\}^2 R \quad \text{or} \quad p(t) = \{e(t)\}^2/R$$

or

$$p(t) = i(t) \times e(t).$$

Mean power may then be written from eq. (1)

$$\text{mean power} = \frac{1}{T}\int_0^T \{i(t)\}^2 R\,\mathrm{d}t = R\frac{1}{T}\int_0^T \{i(t)\}^2\,\mathrm{d}t.$$

For convenience this is usually written as mean power $= RI^2$ where

$$I = \left[\frac{1}{T}\int_0^T \{i(t)\}^2\,\mathrm{d}t\right]^{\frac{1}{2}}$$

is called the root-mean-square value of the current or more simply, the rms value of the current. Similarly

$$\text{mean power} = \frac{1}{R}\left[\frac{1}{T}\int_0^T \{e(t)\}^2\,\mathrm{d}t\right] = E^2/R$$

where

$$E = \left[\frac{1}{T}\int_0^T \{e(t)\}^2\,\mathrm{d}t\right]^{\frac{1}{2}}$$

is called the root-mean-square value of the voltage or more simply, the rms value of the voltage.

What is meant by the term 'rms power'?

This term, which is creeping into the language, is almost always used erroneously. What is commonly meant is *mean power* and that is the term we shall use when we wish to discuss quantities such as E^2/R, I^2R where E, I are rms quantities. If the current and voltage are sinusoidal, mean power may also be written $EI \cos \phi$, where $\cos \phi$ is the power factor of the circuit.

To answer the question 'What is rms power?' let us write an expression for the rms value of any time-dependent quantity, say $x(t)$

$$X = \sqrt{\left[\frac{1}{T}\int_0^T \{x(t)\}^2\,\mathrm{d}t\right]}$$

where X is the rms value of $x(t)$ and where T is any integral multiple of the period of $\{x(t)\}^2$. Similarly

$$P = \sqrt{\left[\frac{1}{T}\int_0^T \{p(t)\}^2\,\mathrm{d}t\right]}$$

where P = rms power. Let us examine this expression for the particular case where a sinusoidal current $\sqrt{2}I \cos \omega t$ flows in a resistor R. Then

$$p(t) = (\sqrt{2}I \cos \omega t)^2 R$$

therefore

$$P = \sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi} 4I^4 \cos^4 \omega t\, R^2\,\mathrm{d}t\right)}$$

which may be written $2\omega t$ $4\omega t$

$$P = \sqrt{\left\{\frac{4I^4R^2}{2\pi}\int_0^{2\pi}\left(\frac{9}{8} + \frac{\cos 2t}{2} + \frac{\cos 4t}{8}\right)\mathrm{d}t\right\}}$$

$$= \frac{3}{\sqrt{2}} I^2 R = \frac{3}{\sqrt{2}}\,(\text{mean power}).$$

Obviously *rms power* is not the same as *mean power* and indeed it is difficult to imagine any physical significance for the concept rms power. *G.J.J.*